Arc and point elasticity

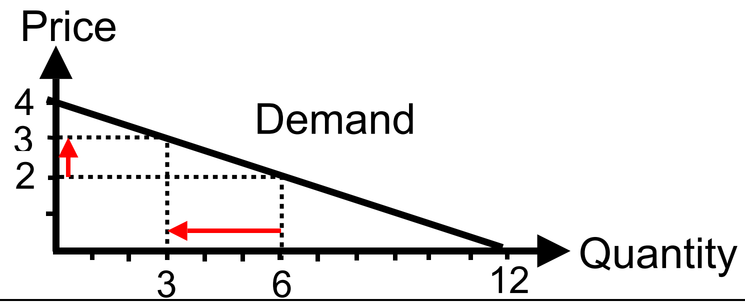
The following explanations refer to the **price elasticity of demand** (e).

1 Formulae

(Q = Quantity; P = Price; P1 and Q1 are initial values, P2 and Q2 are final values.)

Arc elasticity	**Point** elasticity
$e = \frac{\Delta Q}{Q1} : \frac{\Delta P}{P1} = \frac{\Delta Q}{\Delta P} * \frac{P1}{Q1}$ // $\Delta Q = Q2 - Q1$ / $\Delta P = P2 - P1$	$e = \frac{dQ}{Q} : \frac{dP}{P} = \frac{dQ}{dP} * \frac{P}{Q}$ / $\frac{dQ}{dP} = Q'$ (1. derivation)

2 Example of a linear demand function: Q = 12 - 3P

<table>
<tr><th>Arc elasticity: P rises from 2 to 3</th><th>Point elasticity: e at P = 2</th></tr>
<tr><td>


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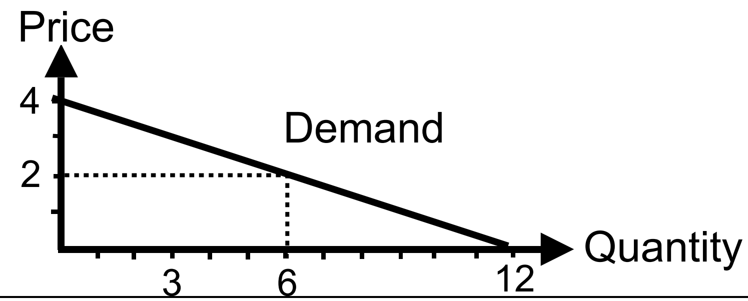
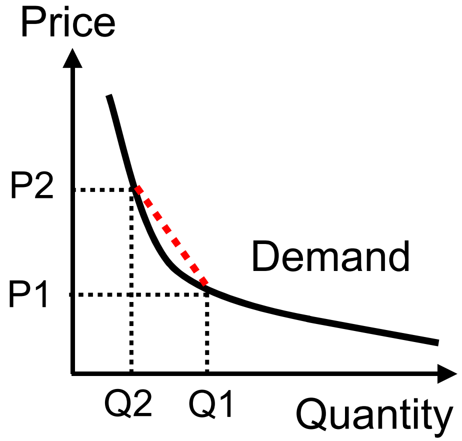
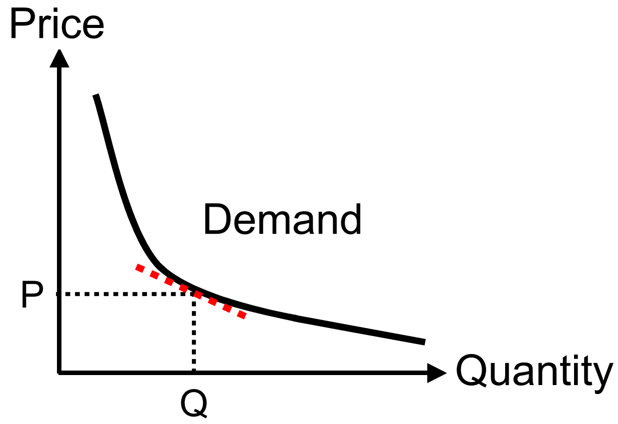


</td></tr>
<tr><td>$e = \frac{\Delta Q}{\Delta P} * \frac{P1}{Q1} = \frac{-3}{1} * \frac{2}{6} = -1$
Average values can be used instead of P1 and Q1, for example '(P1 + P2) : 2' and '(Q1 + Q2) : 2'.</td><td>Q = 12 - 3P → Q' = - 3
$e = \frac{dQ}{dP} * \frac{P}{Q} = -3 * \frac{2}{6} = -1$</td></tr>
<tr><td colspan="2">Remark: In the case of a linear demand function, the two elasticities produce the same result.</td></tr>
</table>

3 Example 1 of a non-linear demand function: in general

Arc elasticity	**Point** elasticity
 The arc elasticity is calculated with the help of 4 variables (Q1, Q2, P1 and P2).	 = tangent / Calculation: Q' >>> Q derived to P

4 Example 2 of a non-linear demand function: $Q = 36 - P^2$

Arc elasticity P falls from 5 to 3	**Point** elasticity e at P = 5
P1 = 5 → $Q1 = 36 - P^2 = 36 - 25 = 11$ P2 = 3 → $Q2 = 36 - P^2 = 36 - 9 = 27$	$Q = 36 - P^2 = 36 - 25 = 11$ $Q' = -2P$
P1 = 5 / P2 = 3 // Q1 = 11 / Q2 = 27	P = 5 / Q = 11 / Q' = - 2P
$e = \frac{\Delta Q}{\Delta P} * \frac{P1}{Q1} = \frac{+16}{-2} * \frac{5}{11} = \frac{80}{-22} = -3.64$	$e = \frac{dQ}{dP} * \frac{P}{Q} = -2P * \frac{5}{11} = -10 * \frac{5}{11} = -\frac{50}{11} = -4.55$

Remarks:

- In the case of a **non-linear** demand function, the two elasticities lead to **different** results.
- Since the price elasticity of demand is usually negative, the minus sign is often omitted.