Microeconomics and mathematics (with answers)

6 Maxima and minima

Steps of optimization:

① Set 1st derivative = 0, then calculate Q.

② Find 2nd derivative:
If 2nd derivative > 0 → Minimum
If 2nd derivative < 0 → Maximum

6.1 Maximize total revenue (TR)

Total revenue = $400Q - 8Q^2$

Find the maximum TR (Q and TR).

6.2 Maximize profit π (π = TR - TC)

Total revenue = $400Q - 8Q^2$
Total cost = $3000 + 60Q$

Find the maximum π (Q and π).

6.3 Maximize total revenue (TR)

Market demand: $P = 12 - \frac{Q}{3}$

Find the maximum total revenue (Q and TR).

6.4 Minimize average cost (AC) and marginal cost (MC)

Average cost = $30 - 1.5Q + 0.05Q^2$

6.41 Find the Q of minimum average cost.
6.42 Find the Q of minimum marginal cost.
6.43 Explain the result of 6.41 in relation to 6.42 (→ relation MC to AC).

6.5 Optimization by a monopolist

The demand function of a monopolist is
$P = 30 - 0.65Q$
and his total cost function is
$TC = 0.5Q^2 + 10Q + 50$

Find the Q which results in the ...

6.51 minimum average cost;
6.52 maximum total revenue;
6.53 maximum profit (π).

6.6	**Minimize marginal cost (MC)** Marginal cost = $0.03Q^3 + 0.01Q^2 - 5Q + 30$ Find the minimum (Q and MC).
6.7	**Maximize profit p (p = TR - TC)** Total revenue = $400Q - 8Q^2$ Total cost = $\frac{1}{3}Q^3 - 2Q^2 + 3Q + 600$ Find the maximum (Q and π).

→ Answers. Click here!

Answers *Microeconomics* and mathematics

6 Maxima and minima

6.1 Maximize total revenue (TR)

- $TR = 400Q - 8Q^2$
  $(TR)' = MR = 400 - 16Q = 0$
  $16Q = 400$
  $Q = 25$
- $(TR)'' = - 16 \rightarrow$ Maximum because $(TR)'' < 0$
- **TR** $= 400*25 - 8*25^2 = 10000 - 5000 =$ **5000**

6.2 Maximize profit p (p = TR - TC)

- $\pi = TR - TC = 400Q - 8Q^2 - 3000 - 60Q = - 8Q^2 + 340Q - 3000$
- $\pi' = - 16Q + 340 = 0$
  $16Q = 340$
  $Q = 21.25$
- $\pi'' = - 16 \rightarrow$ Maximum because $\pi'' < 0$
- $p = - 8*21.25^2 + 340*21.25 - 3000 = - 3612.5 + 7225 - 3000$ **= 612.5**

6.3 Maximize total revenue (TR)

- $P = 12 - \frac{Q}{3}$

  $TR = P*Q = 12Q - \frac{1}{3}Q^2$
- $(TR)' = MR = 12 - \frac{2}{3}Q = 0$

  $\frac{2}{3}Q = 12$

  $Q = 18$
- $(TR)'' = - \frac{2}{3} \rightarrow$ Maximum because $(TR)'' < 0$
- **TR** $= 12*18 - \frac{1}{3}18^2 = 216 - 108 =$ **108**

6.4 Minimize average cost (AC) and marginal cost (MC)

6.41
- $AC = 30 - 1.5Q + 0.05Q^2$
  $(AC)' = - 1.5 + 0.1Q = 0$
  $0.1Q = 1.5$
  $Q = 15$
- $(AC)'' = 0.1 \rightarrow$ Minimum because $(AC)'' > 0$

6.42
- $TC = AC*Q = 30Q - 1.5Q^2 + 0.05Q^3$
  $(TC)' = MC = 30 - 3Q + 0.15Q^2$
  $MC' = -3 + 0.3Q = 0$
  $0.3Q = 3$
  $Q = 10$

6.4 cont.	• $MC'' = 0.3$ → Minimum because $MC'' > 0$ 6.43 The marginal cost curve is crossing the average cost curve from below. Therefore, the minimum quantity of MC is smaller than the minimum quantity of AC.
6.5	**Optimization by a monopolist** 6.51 • $AC = 0.5Q + 10 + \frac{50}{Q}$ $(AC)' = 0.5 - 50Q^{-2} = 0$ $0.5 = 50Q^{-2}$ $0.5Q^{2} = 50$ $Q^{2} = 100$ $\mathbf{Q = 10}$ • $(AC)'' = 100Q^{-3} = \frac{100}{1000} = 0.1$ → Minimum because $(AC)'' > 0$ 6.52 • $TR = P*Q = 30Q - 0.65Q^{2}$ $(TR)' = MR = 30 - 1.3Q = 0$ $1.3Q = 30$ $\mathbf{Q = 23.1}$ • $(TR)'' = -1.3$ → Maximum because $(TR)'' < 0$ 6.53 • $\pi = TR - TC = 30Q - 0.65Q^{2} - 0.5Q^{2} - 10Q - 50 = -1.15Q^{2} + 20Q - 50$ $\pi' = -2.3Q + 20 = 0$ $2.3Q = 20$ $\mathbf{Q = 8.7}$ • $\pi'' = -2.3$ → Maximum because $\pi'' < 0$
6.6	**Minimize marginal cost (MC)** • $MC = 0.03Q^{3} + 0.01Q^{2} - 5Q + 30$ $(MC)' = 0.09Q^{2} + 0.02Q - 5 = 0$ $Q = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-0.02 \pm \sqrt{(0.02)^2 + 4 * 0.45}}{0.18}$ $Q1 = \frac{-0.02 + 1.34}{0.18} = 7.3$ [$Q2 = \frac{-0.02 - 1.34}{0.18} < 0$] • $(MC)'' = 0.18Q + 0.02 = 0.18*7.3 + 0.02 = 1.3$ $Q = 7.3$ → $(MC)'' = 1.3$ → Q is a minimum because $(MC)'' > 0$. [Q2 < 0; Q is negative; Q must be positive.] → $\mathbf{Q = 7.3}$ • **MC** $= 0.03*7.3^{3} + 0.01*7.3^{2} - 5*7.3 + 30$ **= 5.7**

6.7 **Maximize profit p (p = TR - TC)**

· $\pi = TR - TC = 400Q - 8Q^2 - \frac{1}{3}Q^3 + 2Q^2 - 3Q - 600$

$= -\frac{1}{3}Q^3 - 6Q^2 + 397Q - 600$

$\pi' = -Q^2 - 12Q + 397 = 0$

$Q = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{12 \pm \sqrt{(-12)^2 + 4 * 397}}{-2} = \frac{12 \pm \sqrt{1732}}{-2}$

$Q1 = \frac{12 - 41.6}{-2} = 14.8$ $[Q2 = \frac{12 + 41.6}{-2} = -26.8 < 0]$

- $\pi'' = -2Q - 12 = -2*14.8 - 12 = -41.6$

  If Q = 14.8 → $\pi'' = -41.6$ → Q1 is a maximum because (TC)" < 0.

  [Q2 < 0; → Q must be positive.]

  → **Q = 14.8**

- $p = -\frac{1}{3}*14.8^3 - 6*14.8^2 + 397*14.8 - 600 =$ **2880.8**

→ Back to questions. Click here!