

5 Cost, revenue and profit

Remarks:

- Q = Quantity

Costs

- TC = Total cost (= AC * Q)
- AC = Average cost (= $\frac{TC}{Q}$)
- MC = Marginal cost [= (TC)']
- FC = Fixed cost
- VC = (Total) variable cost
- AVC = Average variable cost (= $\frac{VC}{Q}$)
- TC = FC + VC

Revenues

- TR = Total revenue (= AR * Q)
- AR = Average revenue (price) (= $\frac{TR}{Q}$)
- MR = Marginal revenue [= (TR)']

Profit = π

- $\pi = TR - TC$

5.1 Total and average cost

Fixed cost = 1200

Average variable cost = 4

5.11 Calculate total cost if 300 units are produced.

5.12 Graph total cost as a function of Q (Q = 0,100,200,...600).

5.13 Graph average cost as a function of Q (Q = 100,200,...600).

5.2 Cost, revenue, profit, break-even point

A firm has fixed cost of 300, variable cost of 10 per unit and sells a unit at the price of 16.

5.21 Graph total cost (FC, VC), total revenue and show profit/loss (Q = 0,10,...100) in the same diagram.

5.22 Calculate the break-even point ($\pi = 0$): Q, cost and revenue

5.23 Calculate the quantity where profit is 60.

5.3 Profit, break-even point

5.31 Express profit ($= TR - TC$) as a function of Q by using additionally:

- Fixed cost (FC)
- Average variable cost (AVC)
- Average revenue (AR)

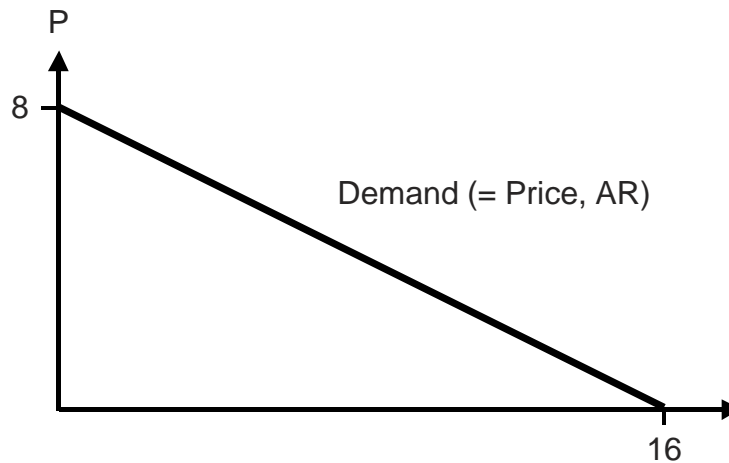
5.32 Calculate profit (loss) by using the the equation obtained in 5.31.

- $FC = 240$
- $AVC = 5$
- $AR (= \text{Price}) = 8$
- $Q = 70$

5.33 Use the equation obtained in 5.31 and the numbers of 5.32 to calculate Q if we target a profit of 60.

5.34 Calculate the break-even point Q using the equation obtained in 5.31 and the numbers of 5.32.

5.4 Demand and revenue



5.41 Find the demand function ($P = \dots$).

5.42 Find the total revenue function.

5.43 Find the marginal revenue function.

5.44 Calculate average revenue, total revenue and marginal revenue if

- $Q = 3$
- $Q = 5$

5.5 Demand and revenue

5.51 Demand: $P = 50 - 5Q$

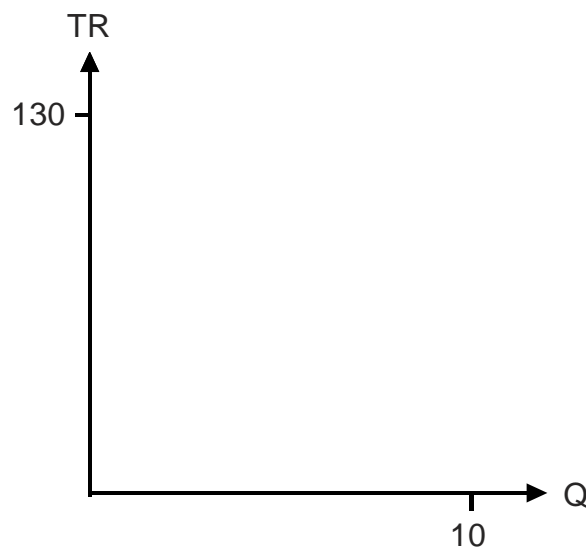
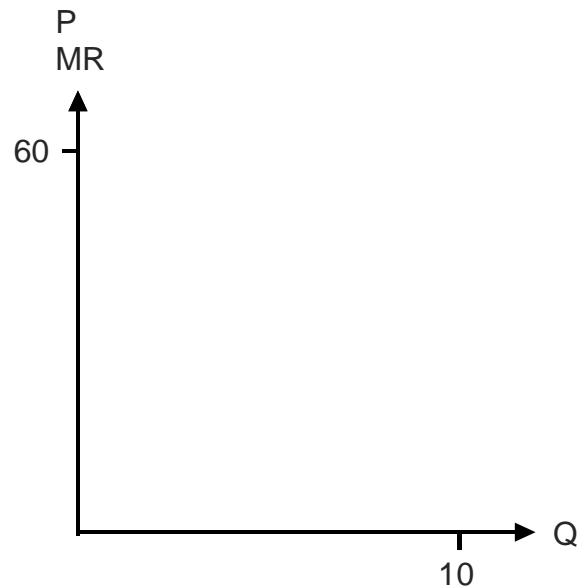
Find the total revenue and the marginal revenue functions.

5.5
cont.

5.52 Calculate marginal revenue if

- $Q = 2$
- $Q = 5$

5.53 Graph demand, marginal revenue and total revenue as follows:



5.54 Which relation exists between $MR = 0$ and TR ?

5.6 Fixed cost, demand and profit

A firm has only fixed cost of 20. Demand: $P = 24 - 3Q$. We suppose that the firm can choose Q .

Which Q results in the highest profit and how much is this profit?

5.7 Cost

Total cost = $5Q^2 + 6Q + 10$

Find the equation for average cost and for marginal cost.

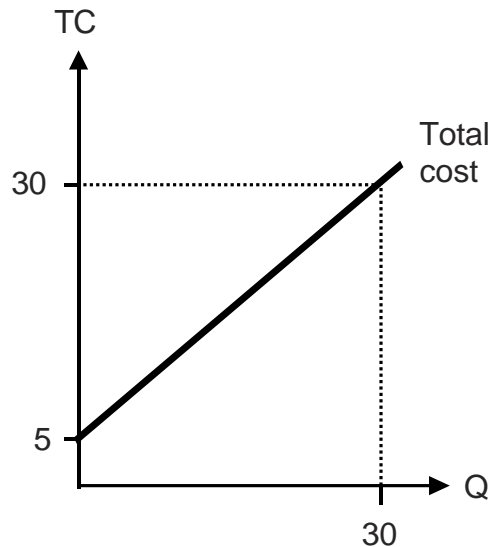
5.8 Cost

$$\text{Average cost} = 6Q + \frac{8}{Q} + 5$$

5.81 Find the equation for marginal cost.

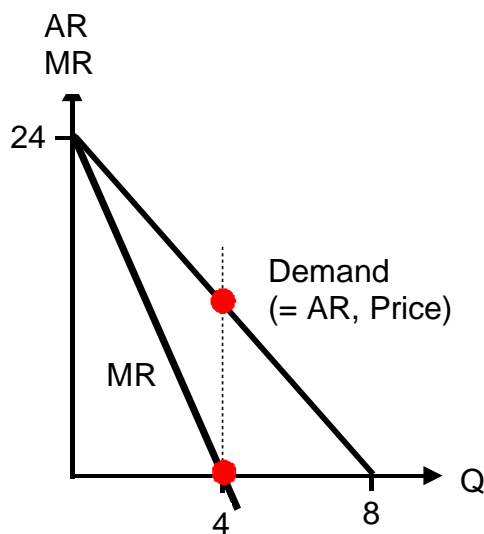
5.82 Output $Q = 30$. Calculate total cost, average cost and marginal cost.

5.83 Another firm wants to buy from us an additional unit for the price of 350. Would you sell the additional unit?

5.9 Cost

5.91 Find the equation for total cost (TC).

5.92 Find the equation for average cost and for marginal cost.

5.10 Marginal revenue and price elasticity of demand (e)

Calculate the price elasticity of demand (e) if $MR = 0$ (at the point $Q = 4$).

$$\text{(Formula for } e = \frac{dQ}{dP} * \frac{P}{Q})$$

| | |
|-------------|--|
| 5.11 | <p>From marginal revenue to total revenue and average revenue</p> <p>Marginal revenue = $20 - 5Q$</p> <p>Find - by integration - the equation for total revenue ($c = 0$), then the equation for average revenue.</p> |
| 5.12 | <p>From marginal cost to total cost and to average cost; fixed and variable cost</p> <p>Marginal cost = $Q^2 + 3Q + 6$</p> <p>5.121 Find - by integration - the equation for total cost.</p> <p>5.122 Which part of total cost is fixed, which part is variable?</p> <p>5.123 Find the equation for average cost.</p> <p>5.124 Calculate total cost, average cost and marginal cost if $Q = 20$. Suppose that $c = 800$.</p> |
| 5.13 | <p>From marginal cost and marginal revenue to total cost and total revenue; profit</p> <p>Marginal cost = $10 + Q^2 - 10Q$ (Fixed cost $c = 50$) Marginal revenue = $20 - Q$ ($c = 0$)</p> <p>5.131 Find - by integration - the equations for total cost and total revenue.</p> <p>5.132 Calculate profit if $Q = 3$.</p> |

→ [Answers. Click here!](#)

Answers *Microeconomics* and mathematics

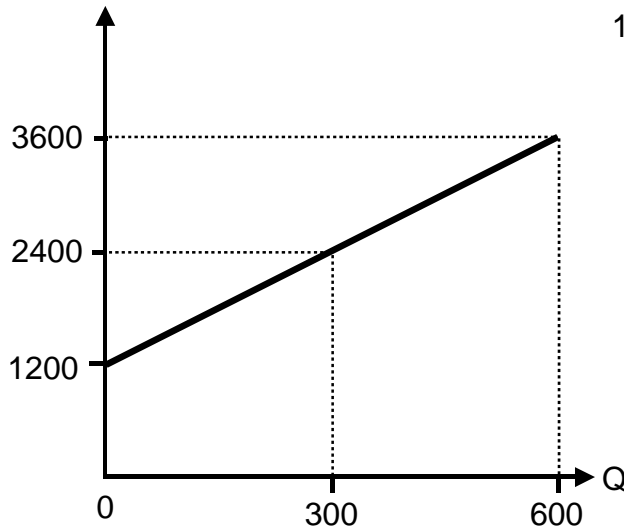
5 Cost, revenue and profit

5.1 Total and average cost

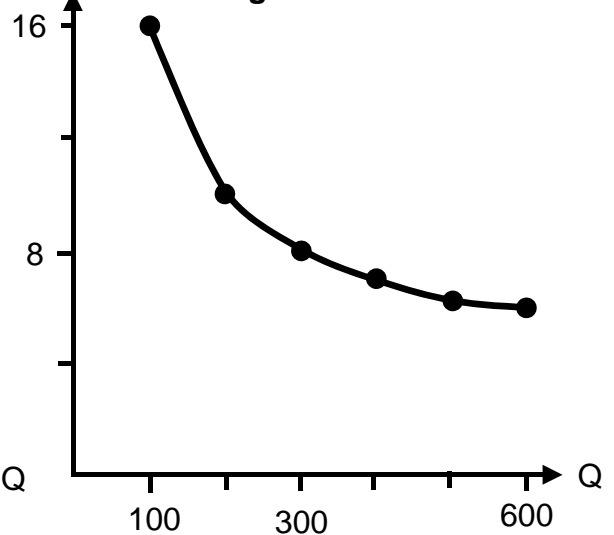
5.11 $TC = 1200 + 4 \cdot 300 = 2400$

5.12/5.13

5.12 Total cost

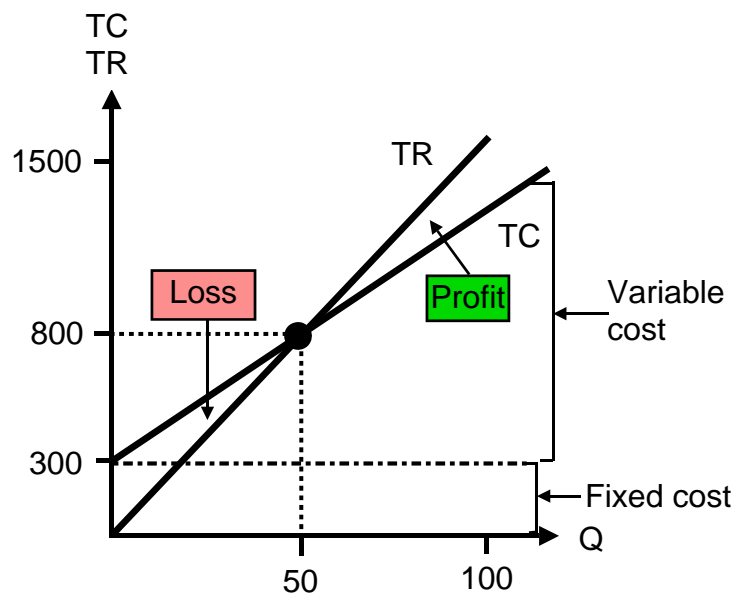


5.13 Average cost



5.2 Cost, revenue, profit, break-even point

5.21 Graph



5.22 Break-even point: $300 + 10Q = 16Q$
 $- 6Q = - 300$
 $Q = 50$ **Cost/revenue: $300 + 10 \cdot 50 = 800$**

5.23 $Q (\pi = 60)$: $300 + 60 + 10Q = 16Q$ $Q = 60$

5.3 Profit, break-even point

$$\begin{aligned}5.31 \quad \pi &= TR - TC \\ \pi &= AR \cdot Q - FC - Q \cdot AVC \\ \pi &= -FC + Q(AR - AVC)\end{aligned}$$

$$5.32 \quad \pi = -240 + 70(8 - 5) = -30 \text{ (} \rightarrow \text{ Loss)}$$

$$\begin{aligned}5.33 \quad 60 &= -240 + Q(8 - 5) \\ -3Q &= -300 \\ Q &= \mathbf{100}\end{aligned}$$

$$\begin{aligned}5.34 \quad \text{Break-even point } (\pi = 0): \\ 0 &= -240 + Q(8 - 5) \\ -3Q &= -240 \\ Q &= \mathbf{80}\end{aligned}$$

5.4 Demand and revenue

$$5.41 \quad P (= AR) = 8 - \frac{1}{2}Q$$

$$5.42 \quad TR = AR \cdot Q = 8Q - \frac{1}{2}Q^2$$

$$5.43 \quad MR = (TR)' = 8 - Q$$

| 5.44 | AR | TR | MR |
|-------|-----|------|----|
| Q = 3 | 6.5 | 19.5 | 5 |
| Q = 5 | 5.5 | 27.5 | 3 |

5.5 Demand and revenue

$$\begin{aligned}5.51 \quad TR &= P \cdot Q = 50Q - 5Q^2 \\ MR &= (TR)' = 50 - 10Q\end{aligned}$$

$$\begin{aligned}5.52 \quad MR \text{ (if } Q = 2): & \quad 50 - 10 \cdot 2 = \mathbf{30} \\ MR \text{ (if } Q = 5): & \quad 50 - 50 = \mathbf{0}\end{aligned}$$

5.53 Page 3

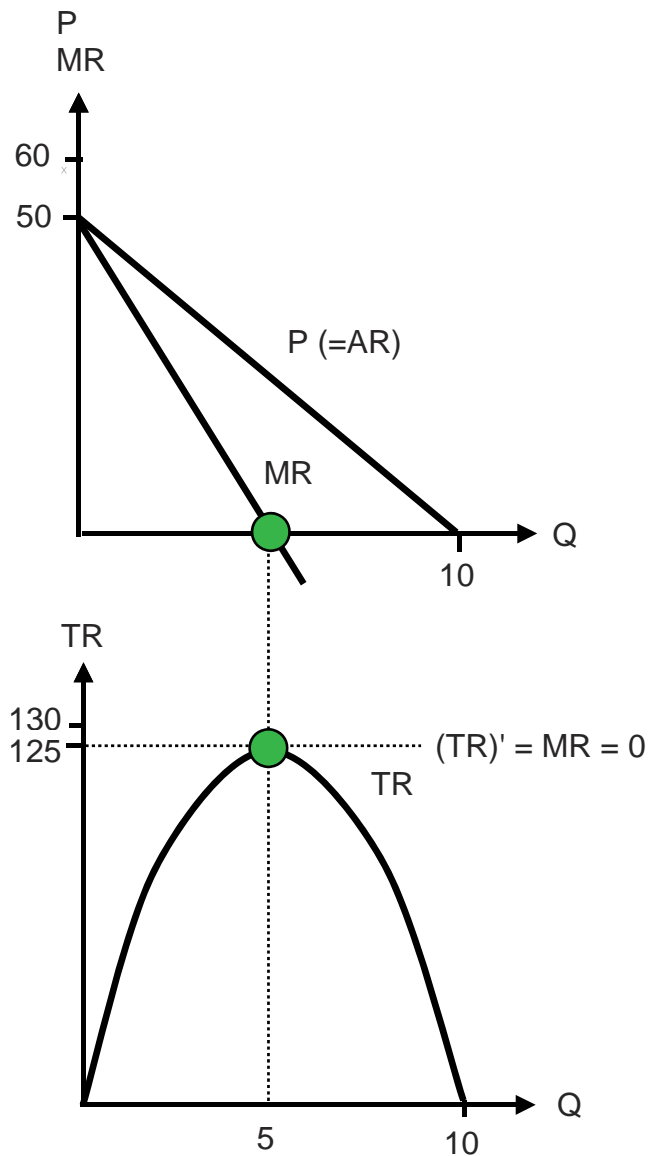
5.54 If $MR = 0$, TR is at its **maximum**.

5.6 Fixed cost, demand and profit

- $TC = 20$
 $TR = Q \cdot P = 24Q - 3Q^2$
 $MR = (TR)' = 24 - 6Q$
If $MR = 0$ then $Q = 4$
- π (if $Q = 4$): $TR - TC = 24 \cdot 4 - 3 \cdot 4^2 - 20 = \mathbf{28}$

5.5
cont.

5.53 Graph demand, marginal revenue and total revenue:



5.7 Cost

- $AC = \frac{TC}{Q} = 5Q + 6 + \frac{10}{Q}$
- $MC = (TC)' = 10Q + 6$

5.8 Cost

5.81 $TC = AC \cdot Q = 6Q^2 + 8 + 5Q$
 $MC = (TC)' = 12Q + 5$

5.82 $TC = 6 \cdot 30^2 + 8 + 5 \cdot 30 = 5558$

$AC = \frac{5558}{30} = 185.3$ (or $AC = 6 \cdot 30 + \frac{8}{30} + 5 = 185.3$)

$MC = 12 \cdot 30 + 5 = 365$

5.83 **No**, I would not sell because $MC > MR (P)$.

| | |
|-------------|---|
| 5.9 | <p>Cost</p> <p>5.91 $TC = 5 + \frac{25}{30}Q = 5 + \frac{5}{6}Q$</p> <p>5.92 $AC = \frac{TC}{Q} = \frac{5}{Q} + \frac{5}{6}$</p> <p>$MC = (TC)' = \frac{5}{6}$</p> |
| 5.10 | <p>Marginal revenue and price elasticity of demand (e)</p> <ul style="list-style-type: none"> $P = 24 - 3Q$ (Demand) $3Q = 24 - P$ $Q = 8 - \frac{1}{3}P$ $\frac{dQ}{dP} = -\frac{1}{3}$ $P = 24 - 3 \cdot 4 = 12$ $\frac{P}{Q} = \frac{12}{4} = 3$ $\frac{dQ}{dP} \cdot \frac{P}{Q} = -\frac{1}{3} \cdot 3 = -1 \quad e = 1$ |
| 5.11 | <p>From marginal revenue to total revenue and average revenue</p> <p>$TR = \int (20 - 5Q)dQ = 20Q - 2.5Q^2$</p> <p>$AR = \frac{TR}{Q} = 20 - 2.5Q$</p> |
| 5.12 | <p>From marginal cost to total cost and to average cost; fixed and variable cost</p> <p>5.121 $TC = \int (Q^2 + 3Q + 6)dQ = \frac{1}{3}Q^3 + \frac{3}{2}Q^2 + 6Q + c$</p> <p>5.122 Fixed part: c Variable part: $\frac{1}{3}Q^3 + \frac{3}{2}Q^2 + 6Q$</p> <p>5.123 $AC = \frac{1}{3}Q^2 + \frac{3}{2}Q + 6 + \frac{c}{Q}$</p> <p>5.124 $TC = \frac{1}{3}20^3 + \frac{3}{2}20^2 + 6 \cdot 20 + 800 = 4187$</p> <p>$AC = \frac{4187}{20} = 209$ (or: $\frac{1}{3}20^2 + \frac{3}{2}20 + 6 + \frac{800}{20} = 209$)</p> <p>$MC = 20^2 + 3 \cdot 20 + 6 = 466$</p> |

5.13 From marginal cost and marginal revenue to total cost and total revenue; profit

$$5.131 \quad \mathbf{TC} = \int (10 + Q^2 - 10Q)dQ = 10Q + \frac{1}{3}Q^3 - 5Q^2 + 50$$

$$\mathbf{TR} = \int (20 - Q)dQ = 20Q - \frac{1}{2}Q^2$$

5.132 Profit $\pi = TR - TC$ ($Q = 3$):

- $TR = 20 \cdot 3 - \frac{1}{2}3^2 = 55.5$
- $TC = 10 \cdot 3 + \frac{1}{3}3^3 - 5 \cdot 3^2 + 50 = 44$
- $\pi = 55.5 - 44 = \mathbf{11.5}$

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